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EXPLANATION IN METAPHYSICS AND BOLZANO'S THEORY OF GROUND AND CONSEQUENCE

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Abstract

In (2006a, 2006b), Benjamin Schnieder criticizes truthmaking as a relation between entities in the world and the truths those entities 'make true'. In (2006b), his criticism exploits a notion of conceptual explanation that is very similar to Bolzano's grounding. In the first part of this paper, I offer an analysis of Bolzano's grounding. I discuss some open problems and argue that Bolzano's grounding is not a systematization of the ordinary notion of 'because' as others have maintained, but of the technical notion of explanatory proof in the context of an axiomatic conception of (proper) science. On the basis of this analysis, in the second part, I offer a critical discussion of Schnieder 2006b's arguments against truthmaking. I conclude that the latter are not very effective from a methodological point of view and that Bolzano's original position fares better in this respect; still, truthmaker theorists will be able to defend truthmaking only at a high price.

1. *Truthmaking, explanation, and Bolzano's grounding*

The notion of explanation has received renewed attention in present-day analytic metaphysics. One example is the debate on the notion of truthmaking as explanatory of truth. Truthmaking is often construed as a form of *grounding*: in particular, the truth of specific statements is said to be grounded

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in some entity in the world playing the role of truthmaker for such statements. For instance, the truth of 'Socrates is pale' is taken to be grounded in e.g. Socrates' paleness, a trope (alternatively, in Socrates' being pale, a fact or state of affairs). This construal of truthmaking as grounding, in turn, has licensed interpretations according to which entities serving as truthmakers provide metaphysical or ontological *explanations* for the truths they are truthmakers of (Simon and Smith 2007[1]). Why is the statement 'Socrates is pale' true? Because there exists a trope of paleness in Socrates (or: because the world contains a fact, Socrates' being pale), which *makes* that statement *true*.

But this is strange, critics say. When asked: why is the statement 'Socrates is pale' true?, one would reply: because Socrates is pale; and if asked, in turn, why is Socrates pale?, one would say: because he's scared to death (cf. Künne 2003: 150). Or one would say: because he's a white guy with skin type I. In other words, answers in terms of truthmaking are a bad kind of explanatory answers to *why*-questions about truth and predication (and indeed about anything else) — or so goes, arguably, the critic's argument. *Causal* explanations are good (e.g. 'Socrates is pale because he's scared to death'), *conceptual* explanations are good (e.g. 'Socrates is pale because he's a white guy with skin type I'), but *metaphysical* explanations — such as "Socrates is pale' is true because there exists a trope of paleness in Socrates' — those are bad.[2]

The debate above takes place within a quite specific discussion on truthmaking in metaphysics. Although it involves a general notion of explanation, i.e. both causal and non-causal, the debate hasn't crossed paths with the vast literature on explanation in philosophy of science. This is not much of a surprise: due to emphasis on natural sciences, physics in particular, philosophy of science tends to centre on causal explanation, while the debate mentioned

[1] "'John exists', 'Socrates is mortal', 'That event is a kissing'. Judgments in this group are true if and only if the entity to which existence is attributed, or of which something essential is predicated, does in fact exist. The existence of that entity yields an *ontological explanation* of the corresponding truth." (Simon and Smith 2007, my emphasis). In the case of standard accidental predications such as 'John is hungry', it is the existence of a quality (or trope) of being hungry (that is, John's) that provides us with an ontological explanation.

[2] This criticism of truthmaking is directed against truthmaking's being explanatory of specific truths such as 'Socrates is pale' (Schnieder 2006b). In this paper, I will concentrate on the latter criticism and disregard objections according to which (trope-theoretical) truthmaking cannot give a unified account of what all truths have in common, *ergo* it can give no explanation of the *concept* of truth (Künne 2003: 148).

above makes appeal to notions of non-causal explanation which philosophy of science tends instead to marginalize (cf. Schnieder 2006b: 38–9).[3] Generally speaking, the terminology and the conceptual apparatus of discussions of explanation in philosophy of science has little in common with the terminology and the conceptual apparatus of the debate in metaphysics I am considering in this paper (see e.g. Salmon's epistemic, modal and ontic conceptions of explanations in Salmon 1998: 63). I think this situation is unfortunate. Before I say more on this, however, I will say something about how I construe the three kinds of explanation mentioned above throughout this paper. Causal explanations are those in which an object in the world in the broadest sense is said to be explained by its cause: 'p because q' is a causal explanation when q describes or states the cause of the effect described or stated by p.[4] Metaphysical explanations here are truthmaker explanations: 'p because q' is a metaphysical explanation when q describes or states a truth (i.e. a true statement or proposition[5]), and p describes or states (the existence of) an object in the world that plays the role of truthmaker for that truth. As to conceptual explanations, there isn't any particularly helpful characterisation readily available in the literature; 'conceptual explanation' seems to be a rather wide umbrella term for explanations based on concepts ('based on concepts' is how I shall construe 'conceptual' throughout the paper). These include all cases of 'p because q'-statements that are true on the basis of other tacitly or previously accepted truths, rules or other statements regulating the connection of p and q in view of the concepts involved in them, and in such a way that p *follows from* q on that basis (but not vice versa). Examples *par excellence* of statements regulating the connection of concepts are definitions. Another example is natural deduction rules.

[3] Forms of non-causal explanation do play an important role in the philosophy of natural sciences other than physics (e.g. functional explanations in biology). However, the discussion of those forms of explanation does not relate to the kind of explanations at issue here; besides, dominant, mainstream trends in philosophy of science tend to favour philosophy of physics.

[4] For the sake of this paper, it does not matter what I mean by 'objects in the world' here (whether they are (bare) particulars, events, states, tropes, facts or bundles of them) — and I do not intend to commit myself to any metaphysical view in particular. The contrast important to me here is that between objects in the world that can enter causal relations as causes or effects, on the one hand, and items serving as truth-bearers that can refer to objects in the world, on the other.

[5] In view of my treatment of Bolzano and to avoid complications on this point, I will henceforth take propositions as truthbearers (which Bolzano takes to be abstract, non-linguistic items serving as meanings of sentences). My 'concepts' will be then most easily construed as constituents of propositions (though one can take them to be the meanings of terms appearing in sentences in use, or to be just meaningful terms).

On the basis of the above:

(Causal)	Socrates is pale because he's scared to death
(Metaphysical)	'Socrates is pale' is true because there exists a trope of paleness in Socrates
(Conceptual)	Socrates is pale because he's a white guy with skin type I.

Importantly, although in all three cases above 'because' is a two-place predicate connecting two propositions, the relata of the relations that are involved in these three cases are all different: two objects, an object and a truth, two truths. (Causal) is true iff there is a causal relation between two objects in the world: Socrates' being scared to death and his paleness. (Metaphysical) is true iff there is a truthmaking relation between an object in the world and a true proposition: Socrates' paleness and 'Socrates is pale'. Finally, (Conceptual) is true iff true proposition q follows from true proposition p on the basis of at least a third proposition ruling in an appropriate manner the connection of concepts involved in p and q. For example, 'Socrates is pale' (a truth) follows from 'Socrates is a white guy with skin type I' (another truth) because the concept of paleness and that of skin type I are appropriately related in a third truth, say 'human skin type I according to Fitzpatrick's scale is mostly pale in colour'.

As I said, the debate in metaphysics involving the three notions at issue, causal and non-causal (i.e. metaphysical and conceptual), does not cross the debate on explanation in the philosophy of science. I also said that I find this unfortunate. The reason is that no matter how pluralistic one wants to be about explanation, a general account of explanation *tout court* seems to be called for. By this I do not mean that we should strive towards a single model or theory of explanation, be it causal or not; I mean that we should strive towards a unified, maximally broad *discussion* of what we (are prepared to correctly) call by one and the same name: 'explanation'. I see three important, interconnected reasons for this. First, mathematical explanations are non-causal. Surely mathematics is worth its name as a science as much as physics is? If so, explanation in mathematics is a legitimate form of scientific explanation as much as explanation in physics is. What is their common genus then, what grounds their acceptability as *both forms of explanation*? Secondly, mathematical explanations come in two kinds (see Mancosu 2008). I shall call these two kinds *internal* and *external*. Internal mathematical explanations are explanations *within* pure mathematics. Explanatory proofs within mathematics (e.g. Why do the four angles of every quadrangle taken together equal four right angles?), i.e. proofs of mathematical results which involve only other mathematical results (Because every quadrangle can be divided in two triangles whose angles taken together are equal to the angles of the quadrangle, and the three angles of a triangle are

equal to two right angles) are examples of conceptual explanations in this sense. Instead, external mathematical explanations are explanations in which non-mathematical phenomena (e.g. Why do hive-bee honeycombs have a hexagonal structure?) are partially explained by mathematical findings (because any partition of the plane into regions of equal area has perimeter at least that of the regular hexagonal honeycomb tiling). Thus, external mathematical explanations play an important role in giving explanations in the natural sciences (cf. also Baker 2009). Thirdly, if we excessively restrict the scope of meaning of the term 'explanation' (e.g. to 'causal explanation'), thus renouncing a unified understanding of the notion, we won't be able to rely on *very* many important historical case studies for scientific explanation, including many cases of external mathematical explanations for physics (e.g., Newton's exhibition of the system of the world from mathematical principles). That would hamper our understanding of the past, make us miss important insights, and would mean a separation of philosophy (of science) from its history — a perspective that, witness what I have to say in this paper, I find disastrous.

Mathematical explanations are conceptual explanations, namely connections among propositions resting on the properties of some concepts. This means that explanation in mathematics and conceptual explanation in present-day metaphysics do not just cross paths: they are one of a kind. This paper is an attempt to ground this claim by providing historical evidence that the notion of conceptual explanation in present-day metaphysics comes from Bernard Bolzano's notion of grounding, which was elaborated in order to capture a general notion of scientific explanation or explanatory demonstration, of which mathematical explanations were a paradigmatic example. It is indeed Bolzano's idea of grounding that prompted and heavily inspired the debate in conceptual explanation in analytic metaphysics I recalled above, although this rarely emerges from the literature as clearly as it should — which is a pity, since Bolzano was an extraordinary philosopher-cum-mathematician who deserves to be better known. The debts to Bolzano's ideas on explanation are also clear (and better stated) in the literature in philosophy of mathematics, in Kitcher 1975, Detlefsen 1988 and Mancosu 1999, who calls Bolzano's theory of grounding "the first fully developed attempt to provide an account of mathematical explanation" (Mancosu 1999: 430).

The paper is structured as follows: in Section 1, I present an analysis of Bolzano's views on explanation, i.e. his theory of ground and consequence. In Section 2, on the basis of this analysis, I argue that the aim of Bolzanian grounding is not to capture the ordinary meaning of 'because' in everyday language. In Sections 3 and 4, I defend my position against possible objections. This is important, for it is crucial to my purposes to show that Bolzano's grounding had explicit scientific aims. These sections are a

scholarly contribution to Bolzano research and research on history of logic and axiomatics, especially the history of (logical) consequence: I argue that Bolzano wished to reduce grounding to a form of derivability, though he could not, because of obstacles largely technical in nature. In Section 5, I will discuss and evaluate the use made of Bolzano's ideas in the debate on truthmaking as metaphysical explanation.

2. *Bolzano on explanation*

Of the three kinds of explanation we saw above, Bolzano accepts in fact only one: conceptual explanation. Bolzano's views on conceptual explanation in his main work, the monumental *Wissenschaftslehre* (Bolzano 1837, henceforth: WL), come down to his views on grounding (*Abfolge*), a concept whose importance for Bolzano's philosophy is hard to overestimate.[6] Grounding is a relation holding among propositions, not facts, events, substances, or anything else we might take propositions to be about in the most basic and straightforward cases. Only propositions can be grounds (*Gründe*) and consequences (*Folgen*). In this respect grounding is similar to derivability (*Ableitbarkeit*), which corresponds nearly to our notion of (logical) consequence. Very roughly, a proposition p is derivable from q for Bolzano iff whenever q is true, then p is also true (for 'whenever' read 'if, for some admissible variations of some parts of q,'). Although both grounding and derivability can be said to capture the idea of a proposition(s) (objectively) *following from* (an)other proposition(s), they differ in four respects: grounding is a relation which is irreflexive, intransitive, asymmetric, and holds only between *true* propositions; none of this holds for derivability: derivability holds also among false propositions and is reflexive, transitive, and not asymmetric (and neither it is symmetric, nor antisymmetric, i.e. in some cases, p is derivable from q and vice versa; in these cases, p and q are said to be *equivalent*).[7] Consider:

(i) Well-functioning thermometers are higher in summer than in winter (p) because it is warmer in summer than in winter (q) (cf. WL§162, I 192)

[6] In Bolzano 1851/1975: 39, the notion of grounding is listed as being as important as those of derivability, concept, and intuition.

[7] For a reconstruction of grounding and for the difference between grounding, derivability, causality and epistemic reason, a relation between judgments, see Tatzel 2002. For an introduction to Bolzano's logic, philosophy and the basic notions of his work, see Morscher 2008, Sebestik 2008.

(i) is true, but its converse is not: it is not *because* well-functioning thermometers are higher in summer than in winter (q) that it is warmer in summer than in winter (p); it is q to ground p, like (i) says, not the other way around. However, propositions p and q are inter-derivable, i.e. equivalent.[8] The same holds for

(ii) Every pair of circles, one described around the centre a, the other around b, both with radius ab and lying in one and the same plane containing these points must intersect (p) because for every two points a and b there must be a third c such that $ca = cb = ab$ (q) (cf. Bolzano 1833–1841, §13).

The most striking difference between grounding and derivability is perhaps intransitivity. By 'grounding', Bolzano means *immediate* and *complete* grounding so grounding is, so to speak, strictly and merely dyadic and it is *unique*: there cannot be more than a single consequence for each ground and vice versa, there is only a single ground for each single consequence (WL§206). So, the consequence of a consequence is no consequence of a ground. But ground and consequence can be *collections* of propositions; indeed, consequences are *always* collections of propositions because among the consequences of a proposition p there is always the proposition: p is true. Truths which are parts of the collections making up the ground and the consequence in a grounding relation are called 'partial grounds' and 'partial consequences'. In the terms in which I put conceptual grounding, the connection of concepts in p and q is regulated by a third truth; on the basis of what we have seen thus far, the third truth will have to be part of the complete ground of p (in some cases it will be identical with it).

Why does Bolzano need grounding in addition to derivability and why is grounding such an important notion for him? This is an interesting question that is hardly dealt with satisfactorily in the literature. Both elements are due to his general conception of science, in any case his ideal of *a priori* (or conceptual, as I shall say, following Bolzano) sciences such as mathematics. Let's consider this in some detail.

In his pious life, Bolzano took up two enormous enterprises: the creation of a new logic that had to be adequate for the foundation of mathematics and the systematic treatment of all branches of mathematics according to this new logic. The first attempt is published in the *Wissenschaftslehre* ('Theory

[8] Note that Bolzano's *Ableitbarkeit* is not constrained as to which parts of the propositions involved should be varied. In this example, the parts involved in the variation are the parts corresponding to 'summer' and 'winter' (the general form being: 'well-functioning thermometers are higher in x than in y, because it is warmer in x than in y').

of Science', that is, his Logic, 1837) the second is contained in the unpublished *Größenlehre* ('Theory of Magnitudes', that is, his Mathematics). How should we understand 'systematic treatment of all branches of mathematics'? Via grounding: a systematic treatment of mathematics is an ordering of its truths (i.e. true propositions) as a chain of grounds and consequences which is objective, i.e. it is an ordering of mathematical truths as they are *in themselves* (and not to the extent we *know* them or *get to know* them). The task of developing the theory of grounding in all generality belongs to logic. In other words, logic fulfils the methodologically foundational task of a theory of science by affording the proper scientific method and it does this by developing a theory of grounding (Bolzano 1833–1841, §17; WL§1). This foundational task of logic, however, is by no means only geared towards the proper systematization of mathematics. It might seem otherwise because Bolzano calls the method 'mathematical' (see Bolzano 1833–1841). But the method is called 'mathematical' not because it should be applied to mathematics only. Indeed, Bolzano gives examples of grounding in quite different fields of knowledge, including ethics (WL§200, II 348, 4.; Bolzano 1833–1841, §14). The reason the method is called 'mathematical' is because mathematics is the paradigmatic science that can be built according to it, that is, mathematics is a *proper* science (Bolzano 1810: II, §1).

What exactly is a proper science according to Bolzano? Again, we don't know all the details. But here's a rough answer to this. Bolzano was a follower of a traditional axiomatic ideal of science, the ideal which, for example, inspires Euclid's *Elements*.[9] A reconstruction of this ideal in seven requirements is captured in de Jong and Betti 2010's Classical Model (or Ideal) of Science. According to the latter, traditionally a science S is a *proper* or *real* science when:

(1) All propositions and all concepts of S concern a specific set of objects or are about a certain domain of being(s).
(2a) There are in S a number of so-called fundamental concepts.
(2b) All other concepts occurring in S are composed of (or are definable from) these fundamental concepts.
(3a) There are in S a number of so-called fundamental propositions.
(3b) All other propositions of S follow from or are grounded in (or are provable or demonstrable from) these fundamental propositions.
(4) All propositions of S are true.
(5) All propositions of S are universal and necessary in some sense or another.

[9] For more on this, see de Jong 2001.

(6) All propositions of S are known to be true. A non-fundamental proposition is known to be true through its proof in S.
(7) All concepts or terms of S are adequately known. A non-fundamental concept is adequately known through its composition (or definition).

Bolzano championed this model both as a philosopher and in his practice as a mathematician.[10] Put in terms of the ideal captured in (1–7), what Bolzano criticized again and again in mathematicians[11] was not their general adherence to it, for that adherence was widespread. Bolzano's criticism regarded the realization of that ideal *as an ideal of explanation* in mathematical practice. First and foremost, he criticized deviation from this ideal in *strictly scientific expositions* of mathematics. To be more specific, Bolzano advocated a strong construal of (3b) as grounding in the edification of science, and held rigorously and consistently to it, so that a proper science would fulfil (3b) just in case its fundamental propositions related to all other propositions as grounds to consequences (Bolzano 1810: II §2). A strictly scientific exposition is one that matches this structure: it orders the truths of a science in a ground-consequence structure.

The major methodological objections Bolzano expresses towards his fellow mathematicians can be interpreted as linked to this point. Traditional expositions of mathematics, he laments, reverse the order of proofs, which means that they do not seek to give *real* proofs, proofs proceeding from grounds to consequences (*Begründungen*), but at most *certifications* (*Gewißmachungen*, WL§525, 261).[12] To put it in Aristotelian terms, what Bolzano was after were not demonstrations 'of the fact' (ὅτι) but demonstrations of the 'reasoned fact' (διότι):

> At one time something might have seemed superfluous, as when Thales [...] took much trouble to prove that the angles at the base of an isosceles triangle are equal, for this is obvious to common sense. But Thales did not doubt *that* it was so, he only wanted to know *why* the mind makes this necessary judgement. And notice, by drawing out the elements of a hidden argument and making us clearly aware

[10] Given his strong notion of knowledge (*Wissen*), however, it seems that Bolzano cannot be simply said to adhere to (6) without further ado. See on this Konzelmann Ziv 2009.

[11] One of Bolzano's targets was Kästner 1758, a big compendium that went through six editions in 1758–1800 (Johnson 1977: 265).

[12] Cf. the (late) Anti-Euklid, see Mancosu 1999: 436–7; Bolzano 1833–1841, §14, Anm. 1.

> of them, he thereby obtained the key to new truths which were not so clear to common sense. (Bolzano 1804: Preface).[13]

Bolzano implemented the construal of (3b) as grounding in his practice as a mathematician.[14] He also insisted, following the Classical Ideal, that one should give precise definitions (2b), limited to the specific domain of a science (1). One might think at first that such a traditional take on science could not be of help to Bolzano's creativity,[15] but it was in fact Bolzano's insistence on this ideal — starting from the *Betrachtungen* (1804) — and on "the dull essentialist problem of definitions" to lead "Bolzano to break from the bonds of traditional geometry and to explore, or shall we say 'invent', the unknown domain of topology." (Johnson 1977: 263).

According to Bolzano, *proper* sciences are grounding chains of (collections of) truths ((4) above; cf. Bolzano 1833–1841, §14, Anm. 1). Among those truths are fundamental truths (axioms, see 3a), the chain's starting points: these have consequences which they ground, but they have no ground themselves (at least not in that very science; more on this later). Bolzano's enterprise can be described as the search for the adequate construal and application of the requirement that in a proper science the relation among truths constituting that science is grounding, not derivability. And the reason why derivability is not enough, then, is this. Giving a scientific account means providing explanations, that is, establishing a grounding order, settling what grounds what. Properly scientific proofs are thus *only* those proofs where *true* premises are also grounds of their (true) conclusions.[16] These are known as explanatory proofs (Mancosu 1999).[17] Derivability, as such, is way too weak to provide such proofs. As we will see in Section 4, however, I do think that Bolzano genuinely endeavoured to find a constrained notion of derivability strong enough to give him grounding.

[13] See also Bolzano 1810: II §2. On 'fact' vs. 'reasoned fact', see also the note to WL§198, Bolzano 1833–1841, §14 and Mancosu 2008, Section 2.

[14] Cf. Bolzano 1817: 4–6. See also Rusnock 2000: 70 and ff.

[15] For example Waldegg 2001 observes that the Classical Ideal of Science functioned as a Bachelardian 'epistemic obstacle' responsible for Bolzano's conservative attitude towards Euclidean geometry.

[16] Cf. Sebestik 1992: 271. To be precise, since 'ground' is often a collection of propositions, true premises in a specific proof will often be partial grounds, i.e. part of the ground of their consequences.

[17] Elements of Dubucs and Lapointe 2006 seems to imply an alternative interpretation — for reasons of space I will not discuss here whether and how this is the case.

The previous makes us understand one thing: Bolzano's aim in developing his theory of grounding was not to capture our ordinary notion of 'because', but to systematize the technical notion of explanatory proof in the context of an axiomatic conception of (proper) science.

3. *Bolzano's grounding is not a theory of the ordinary concept expressed by 'because'*

The claim I have just made opposes an aspect of Tatzel 2002's analysis of Bolzano's intents. Tatzel criticizes Bolzano's construal of grounding as an intransitive relation apparently on the basis of the assumption that Bolzano's theory aimed at capturing the concept at stake in our ordinary use of 'because':

> Bolzano [...] restricts himself to a very narrow, technical use [...] he uses 'ground' and 'consequence' for *immediate* and *complete* grounds and consequences. [...] I do not agree with Bolzano here. I think that a less strict concept that also captures mediate and partial grounds fits much better with our ordinary use of 'because' (Tatzel 2002: 7).[18]

We use 'because' to express a transitive concept in natural language; *ergo*, Bolzano's theory of grounding does not achieve its aims. Let us agree for the sake of the argument that 'because' expresses a transitive concept in natural language. This granted: I agree that *if* Bolzano's goal had been that of explicating the meaning of 'because' in everyday language, *then* he should have characterised grounding as a transitive relation. The fact is that he doesn't. So, either Bolzano is wrong or we are: either there is something wrong in what Bolzano has to offer us as an explication of the ordinary concept expressed by 'because' or we are wrong in thinking that his goal is

[18] Indeed, Tatzel defines a notion of mediate grounding in his paper and proposes to use this notion to capture the ordinary concept. Note that the reconstruction of Bolzano's theory of grounding in Tatzel's paper is an excellent piece of work, one from which I profited much. If I insist on the differences between Tatzel's interpretation-*cum-evaluation* and mine is exactly because his reconstruction is exemplary — a truly helpful and rich one.

offering us an explication of the ordinary concept expressed by 'because'.[19] I think the latter is the case.

I think that, given Bolzano's aims, it was a sensible thing for him to want the relation of ground and consequence to be intransitive. In this section I shall explain why I think so. Bolzano's goal was to make a proper scientific explanation or proof *unique*,[20] i.e. for every consequence of a science (a collection of theorems), there's one and only one ground for it (an axiom, or a collection of truths including axioms). And, in turn he needed the latter, I maintain, to give an answer to this age-old problem: what makes an axiom an axiom? I shall discuss the point via an analysis of two possible problems of my stance, i.e. that Bolzano's aim was to systematize the technical notion of explanatory proof in the context of an axiomatic conception of science and not to capture our ordinary 'because'.

One might argue that I'm wrong in view of the fact that Bolzano often discusses "the ordinary use of terms", and especially on the basis of paragraphs such as §177, where Bolzano discusses propositions of the form 'A is, because B is'. What Bolzano says in such passages, however, does not show that his technical terms are explications of "the use of ordinary life" (WL§20). Even less is §177 evidence that the *theoretical goal* of Bolzano's theory of grounding is that of capturing the ordinary meaning of 'because'. Bolzano says there that 'A is, because B is' is the linguistic expression through which we express the relation of grounding:

> We say: A is because B is, if we want to say that the ground — the complete or a partial ground of truth A — lies in truth B (§177).

What is going on here? I think the following is going on. First of all, Bolzano *normatively* establishes his technical notion of grounding: in §162 he fixes 'grounding' as an intransitive relation and 'ground' as 'complete ground'. *Then* (§177) he looks *descriptively* at how linguistic expressions of the form 'p because q' fare in this respect and he finds out that they express, in terms of the notion of grounding he has fixed, *either complete or partial* grounding. But Bolzanian grounding is not *either* complete *or* partial: it is complete. So

[19] An anonymous referee has pointed out that my interpretation and Tatzel's interpretation are not incompatible since Bolzano might have two goals. But the two goals are incompatible, for the same relation cannot be both transitive and intransitive. One can accept both kinds of relations, of course, but then they cannot both be primitive. Bolzano's primitive grounding is intransitive. So, we should seek an interpretation which first of all does justice to this. My interpretation attempts to do that, Tatzel's does not: he gives no grounds why Bolzano should at all *want* to have intransitive grounding.

[20] Cf. WL§528, 266; Sebestik 1992: 260; 276.

I do not see how §177 can count as evidence for the claim that Bolzano's grounding aims at capturing 'the everyday concept' expressed by 'because'. As was said, this concept is at most that of partial grounding. If Bolzano wanted to capture that, his (primitive notion of) grounding would be partial grounding, but it is not. It should be clear that I am not denying that Bolzano does engage in discussions about linguistic use, for he obviously does. In fact, it would be strange if he didn't: given Bolzano's emphasis on writing scientific treatises and his attention on how to communicate scientific findings in an apt way, rhetorics and semiotics are important to him. What I deny is that Bolzano's *methodology* is that of looking at how people[21] use words such as 'because' to collect normative evidence for philosophy, i.e. evidence on the basis of which to decide what the concept of grounding *is*. Bolzano is not an ordinary language philosopher.[22] His approach seems to be rather the opposite: if ordinary language conforms to what is useful and sensible to scientific aims, good. If not, well, this is a problem for and with ordinary language — and something writers of science should be keenly aware of. A passage from §280 seems emblematic of Bolzano's attitude:

> A distinction among our representations which is so remarkable that we find ourselves induced to speak about it frequently even in ordinary life is the distinction thanks to which we usually divide them into clear and obscure. Yet in ordinary life one would hardly associate the same concepts with these expressions all the time, and we are, therefore, not only allowed but even obliged to specify their

[21] It is not even clear how we should interpret 'people' here. All German-speaking Czechs? All Czechs? Writers of newspaper articles? Sometimes Bolzano seems to take 'common use' to be the use of his colleague *scientists*, philosophers, or mathematicians in their writings; cf. WL§39, where 'Menschen' equals 'the readers of the *Theory of Science*'.

[22] Tatzel seems to assume this much when he says: "His reason [Bolzano's, A.B.] is that he thinks that what I call 'mediate grounds (consequences)' are not really grounds (consequences) in the proper sense of that word (cf. §§213, 217). *I don't think that Bolzano is right in thinking so, as long as our understanding of 'ground' and 'consequence' is derived from our ordinary use of 'because'*." [...] "I therefore conclude that if *Bolzano's basic assumption, that 'because', in its ordinary language use, brings into play a relation between truths*, is true at all, then the concept of mediate grounding is the best candidate for its meaning." (my emphasis). Tatzel also writes: "There are different possibilities for fixing a concept that prima facie corresponds to the intuitions Bolzano is alluding to." (Tatzel 2002: 7). As I am trying to show, Bolzano is not 'alluding to intuitions' about the ordinary concept expressed by 'because', he has scientific explanation in mind, and, paradigmatically, mathematical explanatory proofs.

> meaning here in the way which is most useful to the purposes of science. (WL§280, III 25).[23]

So, in case of conflict between common use and scientific aims, Bolzano thinks it is *obligatory* to deviate from common use. When he discusses the ordinary use of some terms, Bolzano's aim is often to sweep away the irrelevant, dangerous *secondary presentations* (*Nebenvorstellungen*, Bolzano 1833–1841: §9, 2. Anm. 1) that might be awoken in readers, or, in case of *primitive* technical notions, to introduce them by 'descriptions' (*Verständigungen*) — a technique which he explicitly set out to master as part of a traditional concern on epistemic access to sciences as exposed in textbooks (cf. Bolzano 1833–1841: §9*ff.*). Again, there might be many such expositions: *rigorously scientific* will be only those which match the objective order of grounds and consequences.

Another possible weakness of my stance is the following.

4. *Bolzano's Problem: Is Grounding reducible to Derivability?*

If I am right, how come that, first, in the *Wissenschaftslehre* grounding is a *primitive* concept and, second, that Bolzano gives us so little of his supposed *theory* of grounding that to speak of a 'theory' is almost preposterous? If the interpretation I have sketched in the previous section is correct, the fact that Bolzano gives so little of a theory is puzzling, for much seems to depend on the existence of that theory as such. If it does not exist, my interpretation becomes a lot less attractive.

To this I'd say that it is one thing to adhere to some scientific ideal and to set out to realize it. It is another thing to brilliantly succeed on all fronts. It's uncertainty with the latter thing that bothered Bolzano. Here's how I think things stand. In some parts of the *Wissenschaftslehre*, Bolzano does indeed treat grounding as primitive. He notes that he has not managed to find a definition of grounding, this being due either to failure on his side (due to his *subjective* 'ignorance') or because the concept is really primitive, that is, simple (the task is *objectively* 'impossible'). But Bolzano is genuinely unsure whether grounding is a primitive or can be defined. Note that Bolzanian definitions rely on a compositional theory of concepts: a definition of a concept must reveal the (simpler) *parts* of which the concept is composed:

[23] Cf. also WL§35 (I, 161): "For if we take the word presentation in its proper meaning, necessary for the purpose of science...".

> [By] definitions (*Erklärungen*) [...] I understand here nothing other than propositions which specify whether a certain representation [...] is simple or composed of parts, and in the latter case, of what parts it consists and in what connection. (WL§554, IV 330–1; see also §§350–1, III 397–405; §§555–9, IV 332–350; Bolzano 1833–1841: §9.1).

This means that wondering whether grounding is a primitive notion means wondering whether it is a *simple* concept, i.e. a concept that has no parts. The crux is the question whether grounding can be defined as a kind of derivability.[24] Bolzano says that it seems probable that grounding is a kind of derivability (§200, 347) but can't give a proof (§200, 349) and gives an argument why he can't give a proof (§200, 348–9). But twenty paragraphs later Bolzano gives a tentative definition of grounding on the basis of derivability:

> (*) that order among truths in virtue of which from the smallest amount of simple premises the biggest amount of the remaining truths can be derived [*ableiten lassen*] as mere conclusions (§221).

It is unclear why Bolzano is dissatisfied with this definition. For us, it is quite interesting.

First, the definition tells us that Bolzano reduces explanatory power (of a collection of propositions) to other theoretical values: simplicity and (ockhamian) economy (mind that collections of propositions are at issue here, *not* the objects they are about). It is perhaps worth mentioning in this connection that just before writing down (*) Bolzano clearly points out that a *third* theoretical value is involved: generality. However, there is no mention of generality in (*).[25] It is worth dwelling a little longer on the key concepts of simplicity and generality, both relating to (extensional) qualities of concepts. Simplicity is a mereological notion: a simple object is an object that does not have parts, and this holds for concepts as well: a simple concept is a concept that has no parts. Simplicity relates to the *content* of concepts (*Inhalt*), i.e. the mereological sum of their parts. Generality relates instead to the *extension* of concepts (*Umfang*), i.e. the objects falling under them: a

[24] The notion relevant here seems the notion of *exact* derivability, cf. Rusnock 2000: 149–153.

[25] Indeed, in a strictly scientific ordering, simplicity overrules generality: "The simpler truth must be stated in advance of the more complex and, here there is equal complexity, the more general must always be stated before the more particular." Bolzano 1833–1841: 17 (69v), cf. also Sebestik 1992: 275. For the simplicity requirement for axioms in the early Bolzano, cf. Sebestik 1992: 276.

concept is more general the greater the cardinality of its extension (questions of different mathematical infinities aside). I said above that Bolzano criticized expositions of mathematics in his predecessors for reversing grounds and consequences; now we know that this is the case when more complex truths are put before less complex ones and less general truths precede more general ones. A grounding chain is thus a chain "whose complexity grows the greater the distance from its origins, at the same time as its generality diminishes." (Waldegg 2001). This idea can be found everywhere in Bolzano's writings, starting from his early mathematical works.[26] Note that Bolzano reduces causal explanations to relations of grounding. I will come back to causality later.

Secondly, definition (*) fulfils the *desiderata*, for only truths are involved; asymmetry and irreflexivity are secured by a (degree of) simplicity constraint; in turn, the latter is reduced to proper parthood since Bolzano's theory of concepts, as we saw, is compositional; intransitivity is granted by the two-fold partition of a collection of truths making up a science in such a way that axioms (The Ground) ground their theorems (The Consequence), and there is nothing left for theorems to ground (except 'unsubstantial' propositions of the form 'theorem p is true'). Now recall the axiomatic context I mentioned in the previous section. Read from that perspective, the quote enables us to define axioms straightforwardly as just those *simple truths* from which, taken together, the *highest number* of other truths are *derivable*. (The concepts italicized here are defined in the previous parts of the *Wissenschaftslehre*.)

Both points above combine well with the circumstance that Bolzanian axioms do not have any mysterious property of evidence that makes them axioms:

> [I]t will easily be seen that [evidence, ab] is very little suited for providing a firm basis for the classification of all truths into two classes, that is, into axioms and theorems. (Bolzano 1810, B §10).[27]

We can now say why it makes sense for Bolzano to have *intransitive* grounding: it makes sense because all and only proper grounds of a science are in fact axioms, and the consequences are the theorems (in specific proofs the ground might contain other truths — *Hilfswahrheiten* — next to theorems). So, the way in which Bolzano considers the axioms of a science,

[26] See Part II of Bolzano 1804 and Bolzano 1817: 4–6 (*Vorrede*). Cf. also Folta 1981: 25. Bolzano's graphic representations for proofs are 'grounding trees', cf. WL§220, Tatzel 2001.

[27] Cf. also the first rule in the *Vorrede* of Bolzano 1804 (Folta 1981: 19).

its grounds, is, *mutatis mutandis*, the way in which Bolzano's posterity will look at an *axiom system* for a particular theory, say a system such as Frege's *Grundgesetze*, Russell & Whitehead's *Principia* or Leśniewski's systems of Protothetics, Ontology and Mereology.[28] On the one hand, we have the axioms; on the other hand, everything following. The major difference with later system-builders of axiomatic *a priori* deductive sciences like Frege and Leśniewski is Bolzano's conviction that grounding had to be unique (Bolzano 1833–1841: §14); this excludes alternative axiom systems. I shall come back to this at the end of the paper.

To sum up: although Bolzano does not take up the tentative definition of grounding at §221 as the correct one, that definition makes perfect sense in light of his ideal of science and of his system. It remains to be shown where his doubts come from (given that, as I maintain, Bolzano's aim is not capturing the ordinary use of 'because'). It should be kept in mind at any case that the whole issue has deep ramifications, and is far too big for this paper. I will thus limit myself to some preliminary remarks as to what I think a deeper analysis should turn to.

Sebestik points out that derivability is a formal relation, while grounding is a material one: it depends on "the particular character of the ideas involved" (Sebestik 1992: 266; WL§200). Let's see this: Bolzano distinguishes between *material* and *formal* grounding. The second notion, formal grounding, is grounding between propositions which are also derivable and it is defined as a special kind of derivability on the basis of the first notion, (undefined) material grounding (§162, 193; §168, 207). The best way to put the problem, then, seems this:

(Bolzano's problem) Is material grounding reducible to formal grounding?

Or, to be more precise, is grounding — grounding *tout court* — definable as a special kind of derivability? The problem is, again, largely internal to Bolzano's system and his view of axiomatic science. Before I try to say more on this — more than what has already been said in the literature — one thing needs clarification. I said definition (*) enables us to say what axioms are, but I did not say whether 'axioms' here should be understood as Bolzano's *Grundsätze* or as Bolzano's *Grundwahrheiten*. The former are specific axioms of a science (§420) and are indemonstrable only with respect to that science, but not absolutely indemonstrable; the latter instead are common, absolutely indemonstrable axioms (§214; §486). This means,

[28] Mancosu 1999: 436 finds this unsatisfactory, but I am unsure *exactly* why. It would be unsatisfactory, I take, if Bolzano wanted, like some before him, axioms to possess some intrinsic, ontological or epistemologically constrained qualities next to simplicity and generality, such as, as I mentioned, evidence.

as we saw, that *Grundwahrheiten* are absolutely simple (i.e. composed of absolutely simple concepts, §350, III 402), while *Grundsätze* are relatively simple: their ground could lay in a simpler truth of a more general science. If we take Bolzano literally, the simple truths in (*) are *Grundwahrheiten*.

Back to Bolzano's problem. According to Sebestik 1992: 265–6 — so to speak: I am adjusting things to my reformulation — the roots of Bolzano's problem are causality and an issue raised in the argument in WL§200. Let's see the latter first. In WL§200 Bolzano argues that the answer to the problem is no by means of a surprising counterexample. Take a *practical* truth of the form

(p) One ought to do A

which formally grounds *all* other *practical* truths such as 'One ought not to lie'.[29] This means that the consequence of (p) contains all the practical truths. Now, if A were impossible, there could be no duty to do A. But this means that (p) has a (partial) ground in the *theoretical* truth

(s) A is possible.

So, the complete ground of (p) includes (s). Call this complete ground (s+). Now, we have no inference rule (*Schlussregel*), Bolzano argues, which allows us to infer (p) from (s+). (Note that for Bolzano all inference rules in logic should be based on (logical) derivability, cf. §223, §260.1, Berg GA 12/3 1988: 10). Why is that? Because none of the truths in (s+) can contain the concept of *Sollen*. If any of them did, it would be a practical truth; however, this is impossible, because we have supposed that *all* practical truths are included in the consequence of (p). We have thus found a case of grounding between *underivable* (collections of) propositions (s+ and p). The upshot is: since we cannot derive any proposition containing some concept (in this case, *Sollen*) from a collection of proposition *not* containing it (in this case, s+), then material grounding is not reducible to formal grounding. This means that (*) cannot hold: grounding cannot be defined as a special kind of derivability.

Bolzano's argument is not particularly easy to follow, and more research is needed to get a good picture of what exactly is going on in this passage, but I suspect two assumptions are at work here. The first rests on the availability of appropriate definitions of concepts involved in derivations such as this one. The appropriate definition of *Sollen*, a concept contained in (p) cannot be obtained from the concepts contained in the ground of (p), namely (s+); this means that (s+) and (p), we can say, are logically unrelated. Recall our characterisation of conceptual explanation in the introduction: some third

[29] An example of a (p)-formed truth is Bolzano's 'highest moral law': *Always choose from all actions that are possible for you the one which, all consequences considered, most advances the welfare of the whole, in whatever parts* (RW I, 236; cf. §447, WL IV 119), cf. Morscher 2008, 6.2.

truth must be available which connects the concepts involved in the propositions standing in a grounding relation. In this case, the truth at issue is, arguably, something like 'There cannot be a duty to do A unless A is possible'. Now suppose that the third truth at issue were instead of the form 'There is a duty to do A iff A is possible and...', where the right-hand side is a conjunction of all and only the *theoretical* propositions giving necessary and jointly sufficient conditions for there being a duty to do A (one of which is 'A is possible'). This would amount to giving a definition of *Sollen* on the basis of *possible* or similar theoretical concepts: would (p), in this case, be formally grounded in (s+)? Arguably, yes, or at least, on the basis of Bolzano 1833–1841: §17, I think that Bolzano would say this. This definition cannot be given; it cannot be put into (s+), the ground of (p), so, grounding is not as a special kind of derivability.[30]

Suppose that we now understand the argument. Well, it is still surprising. For isn't Bolzano asking too much of grounding? Note first of all that (p) above is in fact a(n) (specific) axiom of a science, ethics, from which all consequences of that science are derivable, and which contains a concept primitive in ethics, *Sollen*. Now, why on earth would one require, in order for grounding to be defined as a special kind of derivability, that a concept which is *primitive* in a practical science, *Sollen*, be definable from the concepts of another science, a theoretical one? This is where the second assumption comes in. Recall the difference I mentioned above between *internal* and *external* explanation. Let's say that 'internal conceptual explanations' are explanatory relations among *explanandum* p and *explanans* q resting on connections of concepts belonging exclusively to a certain science S; 'external conceptual explanations' are explanatory relations among p and q where p and q are such that the concepts involved in p belong to S, whereas at least some of the concepts of q belong to a second science S', and the third truth at issue connects concepts belonging both to S and S'. What seems to me clear from the argument (and gone unnoticed so far) is that Bolzano does not distinguish between internal and external explanation and thinks of grounding as subsuming *both*. (Note that the picture just sketched is consistent with

[30] An alternative reading of the argument is possible, according to which the availability of a definition of *Sollen* in purely theoretical terms does not solve the difficulty Bolzano points at. The truth which is the definition of *Sollen*, one can argue, cannot be a partial ground of (p) (so it is not contained in (s+)) because the definition still contains *Sollen* in the *definiendum*; and since all propositions containing this notion are partial consequences of p, the definition cannot be contained in the ground of (p), i.e. (s+): ergo, no formal grounding of (p) from (s+) is possible. Which reading is correct depends on the role and the significance that Bolzano assigns to definitions in a grounding structure, and I must leave this very interesting issue open for further research. Whichever reading is correct, however, Bolzano seems still to ask too much of grounding and the problem I discuss below in connection with the second assumption remains.

taking the notion of axioms definable from (*) to be *Grundwahrheiten*.) This I see for now as a hypothesis, though one with enormous implications if true — one which explains Bolzano's difficulties quite well. A major implication is that, arguably, Bolzano accepts a humongous grounding chain common to all sciences, with *Grundwahrheiten* (common axioms) at the beginning. Recall now the Classical Ideal of Science with its (1–7) requirements introduced in Section 1: put in terms of (1–7), *not only* are there many sciences, all constructed according to (1–7), *each* with their own grounding chain of *Grundsätze* (specific axioms) and theorems — say, different internal grounding chains; there is *also* an external grounding chain of common *Grundwahrheiten* grounding the more and more specific *Grundsätze* of the different sciences. (p) is an example of *Grundsatz*: for grounding to be formal grounding, that *Grundsatz* should be derivable from some other, simpler, more general *Grundsätze* (possibly identical with *Grundwahrheiten*). But how is that possible? If this is what Bolzano wanted grounding to accomplish, then the whole idea is doomed. No wonder it is extremely difficult to find examples of *Grundwahrheiten*. The reason why the whole idea is doomed is that for Bolzano whether a proposition grounds another depends on the unique definability/decomposability of the (more complex) concept-parts contained in the consequence in terms of the (simpler) concept-parts contained in the ground.[31] While it is in principle possible to have the idea work for *internal* grounding, it is impossible to have it work for *external* grounding, simply because, like in the example, a *Grundsatz* of a science S will contain at least one concept which is *undefinable* in a (more general) science S': S would *presuppose*, *use* the concepts of S', whereas in S' at least one new primitive concept will appear that could not be *defined* in S'.[32] The 'third truths' of external conceptual explanations cannot be definitions; they must be something else.

[31] Bolzano believes that there are simple concepts, but when it comes to saying which concepts are simple, he is cautious: he often says things like 'probably' simple are concepts such as [something], [has], [existence]; indeed, [Something has existence] is a (rare example of) *Grundwahrheit* (§214).

[32] I cannot add much here, but note that this can be seen clearly by looking at the further development of the notion of grounding in the history of logic: it got split in two. For instance, one can compare Bolzano's internal grounding with deducibility within an axiomatic Leśniewskian system and Bolzano's external grounding with Leśniewski's idea of presupposition among axiomatic systems, and see that what Bolzano wanted to achieve must be achieved in another way, and that the two notions must be kept separate. The details of this comparison, including the relationship with the Aristotelian prohibition on kind crossing still awaits elaboration; this is, again, not the place to do this. On Leśniewski see Betti 2009, Betti 2010.

I'd cautiously want to conclude from the above that, barring the problem just sketched with external grounding, (*) is Bolzano's genuine definition of (internal) grounding.

Wait. What about causality? Here I will limit myself to a remark. As I mentioned, Bolzano reduces it to grounding. This seems an obvious obstacle to Bolzano's project since causality cannot be construed logically (Sebestik 1992: 265). But causality does not seem a problem from Bolzano's point of view. For Bolzano A causes B iff the proposition: A exists, grounds the proposition: B exists. (Here A is a force or a quality of a (collection of) substances, or collections of forces or qualities, and existing objects are real spatio-temporal things being either causes or effects). So, like it or not, for Bolzano the sentence: 'God is the cause of the world' *in fact* expresses a relation of grounding between the truth that God exists and the truth that the world exists (§198). These two propositions can occupy a place in a grounding chain belonging to a science (theology) to the extent that the analysis of their concepts allows it. Bolzano holds that purely conceptual sciences are proper sciences, namely, roughly, sciences which contain only purely conceptual truths and can be built according to his construal of the (1–7) model above (WL§525, IV 261), which also means that grounding in the sense of (*) is applicable to them. Purely conceptual truths contain as parts only concepts, and do not contain any intuitions, which are certain simple and single ideas. Since concepts can very well have existing, real objects in their extension there are purely conceptual sciences *whose objects are existing ones* (theology, parts of ethics, the pure part of physics); those sciences can in principle be proper ones as well, with a grounding structure not differing from that of mathematics (see Bolzano after 1830/1966: 208; Mancosu 1999: 437). But, as said, this possibility depends on that of defining all concepts appearing in the propositions of those sciences.

Causality and heterogeneity of application of grounding in different sciences (Sebestik 1992: 261) do not seem to pose problems to the *concept* of grounding then, but to the *status of certain sciences as proper sciences*. They pose such problems because of Bolzano's adherence to the scientific model expressed in sentences (1–7) *and* because of the humongous grounding chain Bolzano wants.

In a nutshell, my hypothesis is that Bolzano was uncertain about (*) *because* he could not reduce material grounding to formal grounding, and that, in turn, he could not reduce material grounding to formal grounding *because* of the following: (1) he did not distinguish internal and external grounding, (2) he lacked, with respect to external grounding, a rigorous way to dispose a number of sciences in relations of subordination or presupposition, and, largely as a consequence of this, (3) he did not know how to show rigorously which (if any) empirical sciences are proper sciences. All three problems are variously connected with requirements on the (degrees of) simplicity and

generality of the truths involved in grounding chains. So, Bolzano's problem with (*) was not the failure to capture our ordinary 'because', but the arrangement of various (conceptual and empirical) sciences into levels while keeping a unique systematic ordering of their truths as grounds and consequences reducible to formal relations.

Now that we know more about grounding, its aims and difficulties, we can address the question of the use that is made, at present, of Bolzano's views. Recently, Bolzano's ideas on grounding have been taken up by Benjamin Schnieder. I wish to consider in particular Schnieder 2006b, a paper in which Schnieder puts forward a criticism of metaphysical grounding, i.e. truthmaking, which in fact draws heavily on Bolzano's theorizations on conceptual explanation. I am going to look into whether the specific use that Schnieder makes in that paper of the latter, on the basis of what we now know about it, is effective against truthmaking. I argue it is not. Yet, I say, the opponents cannot easily save truthmaking.

5. *Bolzano's grounding in present-day metaphysics*

My question from the Introduction was: why are answers in terms of truthmaking bad answers to explanatory questions? Consider again

(Causal)	Socrates is pale because he's scared to death
(Metaphysical)	*'Socrates is pale' is true because there exists a trope of paleness in Socrates
(Conceptual)	Socrates is pale because he's a white guy of skin type I.

Schnieder 2006b defends a view according to which *causal* and *conceptual* explanations are good, but *metaphysical* explanations are bad.[33] So, why does he think that?[34] In this section I shall put forward some critical remarks

[33] Conceptual/Causal explanations such as 'Xanthippe became a widow because Socrates drank hemlock' are also good. *That* conceptual and causal explanations are good: Schnieder 2006b: 31*ff*.; that metaphysical ones are bad: Schnieder 2006b: 39*ff*. Here I concentrate on Schnieder's criticism of truthmaking as explanatory of *predication* and I shall disregard complaints focusing on the linguistic oddity of truthmaking jargon to laymen's ears ("That Jean's singing makes it true that she is singing, that the apple's redness makes it true that the apple is red [...] are linguistic oddities by any ordinary standards.", Schnieder 2006b: 22).

[34] Note that there is an important difference between two forms of metaphysical grounding which I am ignoring. *Propositional* metaphysical grounding is a relation between propositions as expressed in e.g.

(i) 'Socrates is pale' is true because there exists a trope of paleness in Socrates

about the last part of Schnieder 2006b and relate this discussion to what I say in the previous sections. Before I do that, I shall say that I do not intend to defend any systematic point of view about explanation here, and that my criticism of Schnieder is mainly of methodological nature. In fact, I do share Schnieder's criticism of truthmaking — that means I will be mainly playing the devil's advocate here. What I think is that Schnieder's criticism is not effective, but that truthmaker theorists will have hard time rebutting it anyway.

Why are conceptual and causal explanations good and metaphysical explanations bad? At the very minimum, this depends on what should count as an explanation. Let us just agree that an explanatory relation is asymmetric and irreflexive, for the moment disregarding (in)transitivity. What else? We need to put some more substance on that relation if what we want to talk about is *explanation*, and not, say, proper parthood — and if we want to exclude metaphysical explanation, i.e. truthmaking, as Schnieder does. For if we don't, the question will still remain: what is it that detractors object to truthmaking as purportedly providing explanations? What is it about explanation that licenses conceptual and causal explanations, but not metaphysical explanation? One thing in particular we need to know: how should we construe asymmetry? What determines the direction of explanation? What is it, in whatever determines the direction of explanation, that licenses conceptual and causal explanations, but not metaphysical explanation?

In his previous 2006a, Schnieder (rightly) criticized attempts to explicate the notion of truthmaking as a combination of projection and necessitation. Schnieder 2006b explores another option open to truthmaker theorists, namely that of explicating the concept of truthmaking in terms of the connective *because*:

(TM) x is a truth-maker of p $\leftrightarrow_{df}$ p is true, because x exists.

(ii) Socrates is pale because there exists a trope of paleness in Socrates

Non-propositional metaphysical grounding is instead a relation between e.g. a trope (a non-semantic entity) and a truthbearer (a semantic entity) which is true in virtue of the trope; that relation is expressed in phrases like 'Socrates' paleness makes it true that Socrates is pale'. One can also call the latter 'ontic', as Künne does, the former (that is, i. and ii.) 'ontological' grounding or explanation. Here I focus on Schnieder 2006b's criticism of the propositional sort (ontological), as a possible explication of non-propositional (ontic) grounding. Note that a philosopher eschewing ontic grounding is not thereby eschewing *every* form of correspondence or claiming that there is *nothing* in the world truths are *about*. Indeed, Bolzano is an example in case (see Betti under review; Künne 2003: 108–10).

This fixed, Schnieder moves on to criticize truthmaker theorists by arguing that truthmaking so explicated does not deliver the truthmakers they want. Among others via application of an instance of (a generalization of) 'Aristotle's insight', namely

(T) If 'Socrates is pale' is true at all, then 'Socrates is pale' is true because Socrates is pale,

Schnieder gets from

(Metaphysical) *'Socrates is pale' is true because there exists a trope of paleness in Socrates

to

(S1′) Socrates is pale because there exists a trope of paleness in Socrates.

(Note that this step is by no means innocent, but let this pass). Then he considers the two sentences

(S1) *Socrates is pale because Socrates' paleness exists
(S2) Socrates' paleness exists because Socrates is pale.[35]

And now Schnieder says that (S2) is an explanation whereas (S1) is not (they cannot be both explanations, given asymmetry). So, 'TM-theorists have drawn a blank' (Schnieder 2006b: 41).

Now we can ask: Why? Why is (S2) an explanation and (S1) not? On what does this claim rest? To keep (S2) and rule out (S1) we need to justify why we take the correct direction of explanation to be that of (S2). But Schnieder 2006b does not seem to give a straightforward answer to this question. Or does he? Let's see.

Foot-stamping 'because' — Schnieder takes 'because' to express a *primitive* relation between propositions (Schnieder 2006b: 31) since he thinks its conceptual content does not allow for a reductive analysis. However, this is not very convenient. The reason for this not being very convenient is that Schnieder thinks that there are in fact *two* kinds of explanations, conceptual and causal, whereas, as we saw, he does not think that metaphysical explanations are explanations at all. Philosophers accepting metaphysical explanations, however, might just say that these constitute a third, perfectly acceptable 'because': the 'because' of metaphysical explanation. On which basis can we refute them? Perhaps there's some good reason to do so, but I do not think we can reason as follows: we take *because* to express a primitive explanatory relation, *then* we say that there are actually *two* different

kinds of explanatory relations, conceptual and causal, but not a third, metaphysical kind. Can 'because' *really* be primitive in such a situation? Aren't we foot stamping? If there is a lesson metaphysicians can still learn from the (granted, rather decrepit sounding) Classical Ideal of Science (see Section 2 above) it is that the primitives should be chosen with an eye to their intelligibility (although intelligibility is not an explicit requirement in the framework, it somewhat follows from requirement 7). Now, I am not flat-out saying that 'because' in itself is not intelligible. I am saying that the *specific* 'because' that Schnieder takes as primitive is not *by itself as such* intelligible because that notion is supposed to fulfil too many specific theoretical desiderata: it must a) be conceptual, b) not be causal, c) yet leave space for another notion of 'because' which is causal, d) rule out the metaphysical 'because' (it should also, by the way, not be the 'evidential use of because'; the latter use however seems to be excluded by Schnieder, though not in the same way and with the same purpose as the metaphysical 'because'; never mind this further complication). It could perhaps be objected that Schnieder does not take the supposedly primitive notion expressed by 'because' to be that of conceptual explanation only, but to be a broader notion subsuming both conceptual and causal explanation (at any case, he does not take the causal notion to be reducible to the conceptual notion). But this does not seem to make the situation any better, because now that 'because' expresses a causal or conceptual — *but*, handily, not metaphysical — explanation, so we pass from a foot-stamping 'because' to a question-begging one. (Note that I don't think that the metaphysical 'because' is any better in this respect).

But perhaps this conceptual *because* is not, after all, really primitive? In any case, in order to have some claims come out as good explanations and other claims as bad explanations, one ought to say more on the direction of explanation, that is, one ought to give criteria for the asymmetry of 'because'. And indeed, as we shall see, something like a criterion can be distilled from what Schnieder says.

Criteria for the asymmetry of 'because': *simplicity of concepts* — Let us consider again

(S1) *Socrates is pale because Socrates' paleness exists
(S2) Socrates' paleness exists because Socrates is pale.

There is a reason, after all, that Schnieder gives for why he thinks that (S2) is an explanation and (S1) is not. Namely, 'Socrates' paleness' is *conceptually more complex* than 'Socrates is pale'. Here is his general statement:

> (BS1) [...] The direction of conceptual explanations seems to be owed to factors of conceptual complexity and primitiveness; in general, statements involving complex or elaborated concepts are explained in recourse to more primitive concepts. (Schnieder 2006b: 33)

We see that in order to rule out S1 as an explanation, Schnieder says something that, generally speaking, comes very near to the simplicity constraint in Bolzano's tentative definition of grounding. This seems a good occasion for stressing the remarkable debts that Schnieder 2006b's conceptual explanation has to Bolzano's grounding. The basics of Schnieder's conceptual explanation are exactly the same as Bolzano's grounding: first, conceptual explanation (grounding) is a relation holding among two truths, a relation different from derivability (at any case far stronger), and which can be expressed linguistically by 'because'; secondly, it is an irreflexive and asymmetric relation; thirdly, it is a relation taken as primitive, though the direction of explanation is accounted for by means of (degrees of) conceptual simplicity (and, as I suggested, Bolzano has doubts on its primitivity); fourthly, (T), i.e. 'Socrates is pale' is true because Socrates is pale, is *Bolzano*'s reading of a famous passage by Aristotle.[36] Fifthly, BS1 suggests that Schnieder takes up a compositional theory of concepts (meanings), similar enough to Bolzano's theory of concepts; indeed:

> Canonical designators of particularised properties, such as 'Socrates' paleness', are semantically complex expressions, whose meaning is a function of the meaning of their parts and their way of combining. (Schnieder 2006b: 40)

This is also confirmed by what Schnieder says on (T):

> The explanatory force of (T) is comparable to that in the examples of conceptual explanations discussed so far; it is an explanation of a proposition employing a logically elaborate concept, the concept expressed by 'true', by a conceptually simpler proposition. This latter proposition does not employ concepts which enter into an analysis of the concepts expressed by 'true' (Schnieder 2006b: 36).

So, Schnieder supposes, like Bolzano, that

[36] Cf. WL§198, §205; Ibid.: 11; Künne 2003: 150–1; Schnieder 2006: 35.

(S2) Socrates' paleness exists because Socrates is pale

is explanatory *because* it gets the direction of explanation right, which, in turn, means that it gets the order of concepts right: the more complex concepts in the *explanandum* and the simpler concepts in the *explanans*. In our case, this must mean that the concepts involved in 'Socrates' paleness exists' are more complex that the concepts involved in 'Socrates is pale'.

To this there is an extremely close Bolzanian parallel. For Bolzano 'A is b' (more precisely 'A has b') is the basic form of all propositions. Bolzano's translation of (S2) would be

(S2′) The quality of having by Socrates of (his) paleness has existence because Socrates has paleness

In the *explanandum*, the (pure and mixed) concepts of having, has, existence, Socrates and paleness appear; in the *explanans*, only Socrates, has, and paleness appear. Therefore, this is the correct direction of explanation, not that of (S1), although the two truths connected by *because* do not differ in their ontological counterparts and they are interderivable. A similar case is given in *Wissenschaftslehre* §225:

> The relation of A to b is the relation of certain objects to a quality belonging to them. A conclusion that [...] might be seen as an objective consequence of [A has b, A.B.] (§225, 6. 399).

This means that

(i) Socrates is pale

is equivalent to, and is the objective ground of

(ii) the relation of Socrates to paleness is the relation of certain objects to a quality belonging to them.

Which means that (i) is conceptually prior to (ii) and (i) and (ii) are mutually derivable. Moreover, (i) is also equivalent to and the objective ground of our

(iii) Socrates' paleness exists.

There are three elements of Schnieder's position which Bolzano does *not* endorse. I am going to argue that Bolzano is better off, though. We have seen two of them already: the first is that Bolzanian grounding is intransitive, Schnieder's transitive; the second is that Bolzano has a unique kind of

explanation, conceptual explanation, instead of two kinds, conceptual and causal like Schnieder. I will come back to these two elements in a minute. Let's see the third element of difference first. Bolzano would be opposed to the following claim:

BSClaim: *conceptual* complexity can be read off directly from *linguistic* complexity in *natural language*.

My evidence for BSClaim are passages such as these:

> (BS2) Canonical designators of particularised properties, such as 'Socrates' paleness', are semantically complex expressions, whose meaning is a function of the meaning of their parts and their way of combining [A]. Mastery of the rules that govern the formation of such expressions will give rise to an understanding of any combination of a property term, such as 'paleness', with an arbitrary singular term, such as 'Socrates', as long as the terms combined are understood [B]. But this is just to say that such a canonical designator of a trope ['Socrates' paleness', ab] expresses a *logically complex* concept, the grasp of which requires us to relate it to the concepts expressed by the phrase's components, which *will be conceptually more primitive* [C, my emphasis]. Thus we understand 'Socrates' paleness' along the following line: it denotes a particular instance of paleness, existing as a feature of Socrates just in case that he is pale. Generally, we understand an expression of the form "x's F-ness" to denote a particular instance of F-ness, existing as a feature of x just in case that x is F. [D] (Schnieder 2006b: 40)

I am not sure how many would subscribe to BSClaim without seeing the details; nor do I know how the details, that is, a *theory* for the transformations based on degrees of linguistic complexity in [B] would look like. Besides, in [B] and [D] (BS2) suggests a claim which is even stronger than BSClaim, namely that conceptual complexity is a function of our (psychological or epistemic) *mastery* of *linguistic* complexity in natural language:

> [T]ruth is not analysable in terms of the concepts expressed by 'white' and 'snow', *because someone can have a grasp of the concept of truth without knowing anything about snow or colour white* (Schnieder 2006b: 36, my emphasis).

It seems in fact that *this* is the reason why Schnieder assumes that conceptual complexity can be read off directly from linguistic complexity in natural language. The view that what comes first in cognition, subjectively, matches objective lexico-linguistic and conceptual primacy would be rejected by Bolzano even more firmly than BSClaim. As said, Schnieder supposes, like Bolzano, that

(S2) Socrates' paleness exists because Socrates is pale

is explanatory because it gets the direction of explanation right, which, in turn, means that the concepts involved in 'Socrates' paleness exists' are more complex that the concepts involved in 'Socrates is pale'. But unlike Bolzano, Schnieder seems to think that the reason why 'Socrates' paleness' is more complex *is* that *understanding* 'Socrates' paleness exists' requires *understanding* 'Socrates is pale' first. Again, without knowing the details, we cannot judge whether this position is ultimately better than Bolzano's. There is however one observation that can be made against Schnieder's view of explanation. If my analysis of that view is correct, then the following example is problematic:

(S4) This is water because it is H_20.

This is 'the because of *theoretical reduction*' (Künne 2003: 154) (there are many other possible examples, such as 'This is Tuscan kale because it is *Brassica oleracea acephala palmifolia*'). Now compare

(S5) This is H_20 because it is water ('This is *Brassica oleracea acephala palmifolia* because it is Tuscan kale')

One can very well contend that (S4) is correct and (S5) is not. One can also claim that (S4) is a variety of explanation which is a good parallel to (S1), and that this shows that Schnieder's position is unconvincing and unappealing since it means rejecting something that looks like a perfectly good and useful notion of *scientific* explanation — possibly the most interesting we have to look at. Let us first of all suppose that (S4) is a conceptual explanation: it is based on the truth 'the concept of water is the concept of a substance whose chemical composition is H_20'. Now Schnieder claims that the conceptually simpler (in the sense in which *he* seems to take this) explains the conceptually more complex, so we have

Simpler	*is explained by*	*more complex*	*More complex*	*is explained by*	*simpler*
(S1) Socrates is pale	because	(S2) Socrates' paleness exists	(S2) Socrates' paleness exists	because	(S1) Socrates is pale
✓			✗		

But, if so, in the case of (S4) and (S5) we would have

(S4) This is water	because	it is H_20	(S5) This is H_20	because	it is water.
✓??			✗??		

The problem is that the concept of H_20 seems a far more complex concept than that of water. We can bet that we can't really grasp the concept of H_20 if we do not already have a grasp of that of water. But on Schnieder's account, this would mean that, of the two, 'This is H_20 because it is water' is an explanation, whereas 'This is water because it is H_20' is not. This would mean that theoretical reductions are no explanations. And yet it seems that one can hardly say that giving the exact chemical composition of water does *not* explain what water is, while saying that something is water would instead explain H_20, or what H_20 is. Now, I say that one could argue similarly in defense of truthmaking, on the basis that the expression 'a trope of paleness in Socrates' is technical jargon in metaphysics just as expressions like 'H_20' are technical jargon in chemistry:

(S1′) Socrates is pale because there exists a trope of paleness in Socrates.
(S4) This is water because it is H_20.

Metaphysical explanations, one could argue, are metaphysical reductions. And I say that Schnieder's position does not have a means to exclude the metaphysical explanations *ad* (S1′) without abandoning the theoretical reductions *ad* (S4). Note that the chemical composition of water was fixed by Berzelius in 1826, when Bolzano was writing the *Wissenschaftslehre*; assuming that he knew this, he would say, arguably, that 'water' and 'H_20' differ only linguistically, but they express the same concept. So, neither S4 nor S5 are explanations for Bolzano. *His* position is not in trouble. It would be good to get clear on the relationship between metaphysical explanations,

explanations in science, and the kind of 'common sense' explanations revealed by the kind of ordinary language analysis applied by Schnieder. If not, it seems there are only question-begging moves available.

Indeed, Schnieder 2006b's position on metaphysical explanation looks possible only on a level of analysis of a certain, likely question-begging kind, that is, when 'because' is the because of conceptual explanation *and* "x's F-ness" is *defined* as denoting "a particular instance of F-ness, existing as a feature of x just in case that x is F". Sure the following holds *only* when we refuse from the start to acknowledge any kind of explanatory relation other than conceptual or causal:

> [(S1)] presupposes an explanatory relation, where there is none. No causal and no conceptual explanation is given with it; the conceptual explanation which one might deem it to give would invoke logically complex concepts for an explanation of their more primitive components. But this is to turn things upside down; accordingly I conclude that statement [S1, A.B.] is nothing but a pseudo-explanation. Socrates' paleness does not do much; in particular, it does not make it true that Socrates is pale. (Schnieder 2006b: 41)

Now, this connects to the second element of difference between Schnieder and Bolzano. One might say that one is stuck with *some* question-begging things somewhere anyway. Bolzano isn't stuck *here* however. As we have seen, in Schnieder 2006b, causal explanations are of a different kind than conceptual ones while this honour is precluded to metaphysical explanations with no good ground. But again, this problem does not arise for Bolzano's position: since he reduces causal to conceptual explanation, the direction of explanation of the former is brought back to degrees of conceptual complexity (in his sense). This is also why Bolzano has nothing like metaphysical grounding: for him, explanation only made sense as construed as a relation among propositions and their parts, a relation at bottom extensional in the mereological sense (these are the only explanations *by things* you find in Bolzano). But if we do not want to take up this unified picture (and I am not saying we should), then the kind of direction of explanation depends on the reading of 'because': degree of complexity in the conceptual case, perhaps time or whatever else in the causal case (Schnieder 2006b: 33). But, as I already mentioned, if so, how do we rule out, without *ad hoc* moves, a third *kind* of explanation, metaphysical explanation? This weakens the case against S1 in a fundamental way.

The above also suggests a first, possible way-out for truthmaker theorists. At a metaphysical level, the counterpart of both sides of S1 and S2 is the same object: a simple trope (or a fact, or whatever, see Mulligan 1984 et

al, Simon and Smith 2007, Smith 1999). One could simply say that exactly like in causal connection the direction of asymmetry is given by something *else* than conceptual complexity, in metaphysical explanations the direction is also given by something else. The something else could be, surprise surprise, a metaphysical theory of what things are. One would argue that what determines the direction of explanation is, obviously, the primacy of worlds over words (and concepts). However, this is by no means an easy way to go, for going this way means getting clear on what theoretical reductions in metaphysics are. They do not seem to be cases of internal grounding. But are they cases of *external* grounding? If they are, one could go back to believing that metaphysics is the foundation of all special sciences, where the idea of foundation is captured by external grounding and metaphysics is accepted as a *scientific* enterprise. One would have all reasons to be sceptical of this, but I'm afraid that, should Schnieder's objections to the modal construal of truthmaking prevail, and his challenge hold, truthmaker theorists do not have that many alternatives.

The option of treating metaphysics as scientific enterprise connects to the first element of difference between Schnieder and Bolzano: transitivity. As should be clear from the previous sections, Bolzano's convinctions on grounding rested on his views of what a proper science looked like. I have argued that seeing grounding in this context makes it possible for us to see that the notion of grounding Bolzano was *really* after was that of *formal* grounding, i.e. derivability of a specific sort between the truths forming an axiomatic science, and such that grounds are axioms and consequences theorems. And this, I argued, explains why he wanted grounding to be intransitive: the axioms of a science explain its theorems, full stop — there is nothing theorems have to explain. This is also why Bolzano's doubts about the primitivity of material grounding are genuine. If my interpretation is correct, he could not define grounding in terms of derivability due to limitations of fundamentally technical kind. What real reason do we have to keep a *primitive* notion of grounding which Bolzano would have gladly abandoned, if he had had our formal sophistication and knowledge of axiomatic structures? Perhaps part of the reason to insist on the primitiveness comes from Schnieder's concern with capturing the 'ordinary concept' expressed by 'because' (like Tatzel argues that Bolzano was doing). But why this concern at all? Which aims, and which methodology grounds it, and what's the gain? If I am right in my account of Bolzano's aims, then this is not perhaps the most promising way to be inspired by Bolzano. Why keep on insisting on the primitivity of 'because' if there are reasons to think that that notion can be explained as fruitfulness of axiom systems? Suppose we can't dismiss those reasons. There are, then, more successful *versions* of what Bolzano wanted that we can exploit. Not surprisingly, the only worked out, rigorous, and consistent formal

attempts I know to accomplish what Bolzano wanted, due to Leśniewski and his pupil Adolf Lindenbaum, are formal, *axiomatic* deductive theories (one of which, Ontology, is based on 'A is b'-sentences), and a formal account of the simplicity of notions geared to axiomatic systems (see Lindenbaum 1936). This is not what BS2 seems to want, though. The axioms of Leśniewskian systems are, by that account, simple; but they are extremely demanding to *understand* (indeed, the axiom of Protothetics *is* very much not obvious).[37] Personally, I have no difficulties with taking metaphysics to be Leśniewski's axiomatic, classical, fully extensional Mereology. But I am afraid this isn't a popular position, to put it mildly.

There is a second way in which truthmaker theorists could rebut to Schnieder. Following Bolzano, Schnieder supposes that the linguistic/conceptual complexity of 'Socrates' paleness exists' makes it more complex than the predication 'Socrates is pale'. But why should the direction of explanation be based on the degree of complexity of the *concepts* involved in 'Socrates' paleness' rather than, say, on the priority of existential *forms* of assertion over categorical predications? If we don't give any argument, truthmaker theorists could easily invoke a second criterion for conceptual explanation, one that would block Schnieder 2006b's Bolzano-flavoured arguments. For if the criterion to set the direction of explanation would be given by the priority of existential forms of assertion over categorical predications, what truthmaker theorists want to accomplish would be accomplished. If I were a truthmaking adept (*cum* trope theorist), and wanted to play by the same rules as Schnieder's, I would insist on an alternative construal of the direction of explanation given by the fundamentality of existential forms of predication, something along the lines of a Brentanian analysis of judgement. 'Socrates' paleness exists/Socrates' paleness does not exist', I would argue, are not predications, but just linguistic devices to express the absolutely basic form of a (positive or negative) assertion. For consider

> Judgments in this group ['John exists', A.B.] are true if and only if the entity to which existence is attributed [...] does in fact exist. The existence of that entity yields an *ontological explanation* of the corresponding truth. (Simon and Smith 2007, my emphasis).

Now, what if one insisted that the basic form of assertion is '*A's b* exists', while '*A* is *b*' just derivative, on the basis of the fact that the former, not the latter, expresses at best both the structure of our judgements and how things are in the world? This would just amount to a change of hero: exit Bolzano,

[37] On this theme (but not in connection to Leśniewski), see Shapiro 2009.

enter Brentano, Twardowski & co. What would rule *this* out? Schnieder's methodology would not be able to. However, a move like this comes at a high price, for it would mean for truthmaker theorists to exchange their fundamental notion of metaphysical explanation for both conceptual explanation and an analysis of assertion which is quite atypical, in no way part of common lore. Again, I am not sure how many truthmaker theorists would be prepared to do this.

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